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THE SPECTRUM OF COSMIC RAYS ESCAPING FROM RELATIVISTIC SHOCKS

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ABSTRACT

We derive expressions for the time integrated spectrum of Cosmic Rays (CRs) that are accelerated in a decelerating relativistic shock wave and escape ahead of the shock. It is assumed that at any given time the CRs have a power law form, carry a constant fraction of the energy E of the shocked plasma, and escape continuously at the maximal energy attainable. The spectrum of escaping particles is highly sensitive to the instantaneous spectral index due to the fact that the minimal energy, $\varepsilon_{\rm min} \sim \Gamma^2 m_p c^2$ where Γ is the shock Lorentz factor, changes with time. In particular, the escaping spectrum may be considerably harder than the canonical $N(\varepsilon) \propto \varepsilon^{-2}$ spectrum. For a shock expanding into a plasma of density n, a spectral break is expected at the maximal energy attainable at the transition to non relativistic velocities, $\varepsilon \sim 10^{19}(\epsilon_B/0.1)(n/1{\rm cm}^{-3})^{1/6}(E/10^{51}{\rm erg})^{1/3}$eV where ϵ_B is the fraction of the energy flux carried by the magnetic field. If ultra-high energy CRs are generated in decelerating relativistic blast waves arising from the explosion of stellar mass objects, their generation spectrum may therefore be different than the canonical $N(\varepsilon) \propto \varepsilon^{-2}$.

Subject headings: cosmic rays

1. INTRODUCTION

Cosmic rays (CRs) are widely believed to be accelerated in collisionless shock waves in various systems (see e.g. Blandford & Eichler 1987; Axford 1994). In particular, supernova remnant (SNR) shocks propagating subrelativistically are thought to accelerate protons up to $\sim 10^{15}$ eV (see e.g. Ginzburg & Syrovatskii 1969), while non or trans-relativistic internal shocks or ultra-relativistic external shocks of the jets of active galactic nuclei (AGNs, see e.g. Berezinsky 2008) or gamma-ray bursts (GRBs, Waxman 1995; Vietri 1995; Milgrom & Usov 1995) may be the sources of Ultra High Energy CRs (UHECRs) up to the GZK energies of $\sim$ few $\times 10^{20}$ eV.

In cases where the shocked material expands considerably before releasing the CRs, most of the cosmic rays loose most of their energy by adiabatic losses before escaping. Thus, the instantaneous spectrum of CRs at a given time and the integrated spectrum of the CRs escaping from the system may be different.

In this paper, we write down simple analytic expressions for the spectrum of escaping CRs from a relativistic decelerating shock wave assuming that at any given time (a) CRs escape at the maximal energy attainable at that time, (b) the CRs have a power law energy spectrum, and (c) the CRs carry a constant fraction of the shocked thermal plasma. We show that even under these simple, widely acceptable assumptions, the resulting spectrum may be non trivial and in particular significantly harder than the canonical $N(\varepsilon) \propto \varepsilon^{-2}$ spectrum.

In section § 2 we formulate our basic assumptions and give general expressions for the resulting flux. In section § 3 we focus on energy conserving blast waves expanding in a uniform medium. The implications of the results are discussed in § 4. Expressions for the escaping CR spectrum expected in other scenarios, including power law density profiles, radiative shocks, and a jet injection with constant luminosity, are derived in § A.

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2. BASIC MODEL AND ASSUMPTIONS

Consider CRs that are accelerated by an expanding shock wave which is decelerating due to interaction with an external medium.

2.1. *Assumptions*

(i) The basic assumption that we make is that at any given time (or shock radius R), particles escape at the maximal energy $\varepsilon_{\rm max}$ attainable at that time. Specifically, it is assumed that $\varepsilon N_{\rm esc}(\varepsilon)$, the number of CRs ejected within a logarithmic energy interval around ε, is similar to the number of particles in the system, $\varepsilon N(\varepsilon, R)$, at the radius R for which the maximal energy $\varepsilon_{\rm max}$ is equal to ε,

$$\varepsilon N_{\rm esc}(\varepsilon) \sim \varepsilon N(\varepsilon, R|_{\varepsilon_{\rm max}=\varepsilon}). \tag{1}$$

Here and everywhere else in this letter, all quantities are measured in the observer frame. This assumption is expected to be true whenever the acceleration is limited by the finite size of the system (for a recent discussion see e.g. Caprioli et al. 2009). Note that for a relativistic shock with Lorenz factor Γ, only particles that move in the shock propagation direction to within an angular deviation of $1/\Gamma$, can outrun the shock and escape. This does not introduce a further correction however, due to the fact that all particles in the shocked region, including the thermal and accelerated particles, are beamed in the observer frame to an angular separation of $1/\Gamma$ (for examples of the expected angular distribution of CRs in relativistic shocks see e.g. Kirk & Schneider 1987; Bednarz & Ostrowski 1998; Kirk et al. 2000; Keshet & Waxman 2005).

For concreteness we further make the following assumptions.

(ii) At any given radius R, the energy spectrum of CRs

is a power law, $N(\varepsilon) \propto \varepsilon^{-2-x}$ for $\varepsilon_{\min} < \varepsilon < \varepsilon_{\max}$, with $x > 0$ and with total energy $E_{\rm CR}$. The spectrum can be expressed as

$$\varepsilon^2 N(\varepsilon) = x E_{\rm CR} \left(\frac{\varepsilon}{\varepsilon_{\min}}\right)^{-x}, \qquad (2)$$

where we assumed that $(\varepsilon_{\max}/\varepsilon_{\min})^{-x} \ll 1$.

(iii) The minimal, maximal and total cosmic ray energies are power law functions of the radius,

$$\varepsilon_{\min} \propto R^{-\alpha_{\min}}, \quad \varepsilon_{\max} \propto R^{-\alpha_{\max}}, \quad E_{\rm CR} \propto R^{-\alpha_E}. \qquad (3)$$

2.2. *Resulting spectra*

Under the above assumptions, the spectrum of escaped particles is given by

$$\varepsilon^2 N_{\rm esc}(\varepsilon) \propto \varepsilon^{-x_{\rm esc}} \qquad (4)$$

with

$$x_{\rm esc} = x - (\alpha_{\min} x + \alpha_E)/\alpha_{\max}. \qquad (5)$$

For the case $\alpha_{min} = 0$, this reduces to equation (28) of Ohira et al. (2009).

Equation (5) is valid for $x > 0$. We note that for the limiting value, $x = 0$, a logarithmic correction is introduced to the spectrum of escaped CRs, due to the logarithmic dependence of the total CR energy on $\varepsilon_{\min,\max}$, $E_{\rm CR} = \log(\varepsilon_{\max}/\varepsilon_{\min})\varepsilon^2 N(\varepsilon)$. For $x < 0$, the energy carried by the CRs is dominated by the particles with largest energies, $\varepsilon = \varepsilon_{max}$. The resulting escaped spectrum satisfies $x_{\rm esc} = \alpha_E/\alpha_{max}$ and is not sensitive to the form of the instantaneous spectrum.

3. ENERGY CONSERVING BLAST WAVES

We next apply the above general result to simple cases, focusing on a blast wave of fixed energy which starts off ultra-relativistic, and then decelerates to non relativistic velocity. The non relativistic and relativistic stages are analyzed in § 3.1 and § 3.2 respectively. The spectrum resulting from the combination of the two stages is addressed in § 4.

3.1. *Non-relativistic expansion*

Perhaps the simplest case to consider is the case of constant CR minimal energy $\varepsilon_{\min}$ and total energy $E_{\rm CR}$ ($\alpha_E, \alpha_{\min} = 0$). This is expected in the Sedov-Taylor phase of non relativistic blast waves. The energy in CRs is dominated by the relativistic CRs, $\varepsilon > m_p c^2$, and is commonly assumed to be a constant fraction f of the total energy E in the system. Hence $\varepsilon_{\min} \sim m_p c^2$ and $E_{\rm CR} = fE$, both independent of the shock radius. Eq. (5) implies that the escaped spectrum is the same as the instantaneous spectrum,

$$x_{\rm esc} = x, \quad \text{i.e.} \quad N_{\rm esc}(\varepsilon) \propto \varepsilon^{-2-x}. \qquad (6)$$

This result can be obtained directly from Eq. (1) by noting that the entire spectrum, up to the maximal energy $\varepsilon_{\max}(R)$, is independent of radius.

3.2. *Ultra-relativistic expansion*

We next consider the case of ultra-relativistic expansion. Below and in § A we make the following assumptions in addition to assumptions (i)-(iii) above:

(iv) The minimal CR energy is the energy of the thermal particles,

$$\varepsilon_{\min} \sim \varepsilon_{\rm th} \sim \Gamma^2 m_p c^2; \qquad (7)$$

(v) The CR pressure is a radius independent fraction $f_{\rm CR}$ of the momentum flux in the shock frame, $p_{\rm CR} \propto \Gamma^2 \rho$, implying

$$E_{\rm CR} \propto R^3 \Gamma^2 \rho; \qquad (8)$$

(vi) The maximal CR energy is the maximal energy of CRs that are confined by a fluid rest frame magnetic field $B_{\rm rest}$ (equivalent to Diffusive Shock Acceleration in the Bohm limit),

$$\varepsilon_{\max} \sim e E_{\rm obs} \delta R \sim e B_{\rm rest} R \propto e(\Gamma \rho^{1/2}) R, \qquad (9)$$

where $E_{\rm obs} \sim \Gamma B_{\rm rest}$ is the electric field in the observer frame corresponding to a magnetic field $B_{\rm rest} \sim (8\pi\Gamma^2\rho c^2 \epsilon_B)^{1/2}$ in the rest frame of the shocked plasma, assumed to carry a constant fraction ϵ_B of the momentum flux, and $\delta R \sim R/\Gamma$ is the maximal distance that a cosmic ray can propagate along the electric field in the observer frame.

3.2.1. *Ultra-relativistic impulsive expansion*

We next consider a Blandford- McKee (energy conserving; Blandford & McKee 1976) shock expanding into a uniform medium. By assumption $\alpha_E = 0$. Using equations (7)-(9), $\alpha_{\max} = 1/2$ and $\alpha_{\min} = 3$ are obtained. Using (5) we find $x_{\rm esc} = -5x$, or

$$N_{\rm esc}(\varepsilon) \propto \varepsilon^{5x-2}. \qquad (10)$$

4. DISCUSSION

The spectral index of escaped CRs, given by Eq. (10), may be surprisingly hard if the instantaneous spectrum (equ.[2]) is softer than a flat spectrum, i.e. $x > 0$. The basic reason for this is the fact that the minimal CR energy $\varepsilon_{\min} \sim \Gamma^2 m_p c^2$ changes with radius much faster than the maximal energy, implying that at later times (corresponding to lower escaped energies) there is much less CR energy at the escaping, high end of the spectrum.

For example, consider the commonly assumed Diffusive Shock Acceleration (DSA) mechanism (Krymskii 1977; Axford et al. 1977; Bell 1978; Blandford & Ostriker 1978), which for ultra-relativistic shocks with isotropic, small-angle scattering, leads to an instantaneous spectrum $N \propto \varepsilon^{-20/9}$ (Keshet & Waxman 2005; Bednarz & Ostrowski 1998; Kirk et al. 2000). Using Eq. (10), this implies a very hard escaping spectrum, $N_{\rm esc} \propto \varepsilon^{-8/9}$. We emphasize that DSA has not been shown to work based on first principles and even if it does, the resulting spectrum is sensitive to the scattering mechanism (e.g. Keshet & Waxman 2005; Keshet 2006) which is poorly understood. In fact, the instantaneous spectrum does not necessarily need to have a power-law form (e.g. Caprioli et al. 2009; Ohira et al. 2009) or to have a constant spectral index(e.g. Ellison & Double 2004).

Note that an instantaneous spectrum that is flat $N \propto \varepsilon^{-2}$ (e.g. Katz et al. 2007) or harder (e.g. Keshet 2006; Stecker et al. 2007, and others) can be obtained in specific particle acceleration models. These models, however, are different from the possible hard spectrum of escaping particles discussed above. In fact, such hard

instantaneous spectra lead to a flat escaped spectrum $N_{\rm esc}(\varepsilon) \propto \varepsilon^{-2}$ [see discussion following eq. (5)]. The important point is that the escaping spectrum is highly uncertain, very sensitive to the acceleration mechanism, and may be considerably harder than $N_{\rm esc}(\varepsilon) \propto \varepsilon^{-2}$.

Once the shock decelerates to non relativistic speeds, the integrated escaping spectrum changes its form. In the subsequent Sedov-Taylor phase, the minimal CR energy does not change any more, $\varepsilon_{\rm min} \sim m_p c^2$, and the escaping spectrum will be similar to the instantaneous spectrum [see Eq. (6)]. Note that in the extreme case in which CRs carry most of the energy, and assuming they are accelerated by non-linear DSA (for a recent review see Malkov & Drury 2001), the instantaneous spectrum may not be a power law and correspondingly the escaping spectrum is different than that given here (e.g. Caprioli et al. 2009; Ohira et al. 2009). In any case, a significant break is expected in the escaping spectrum at an energy ε_* equal to the maximal energy achievable at the transition from relativistic to non-relativistic shock velocities. Using Eq. (9) for $\Gamma\beta \sim 1$, the break is expected at

$$\begin{aligned} \varepsilon_* &\sim eB_{\rm rest}R \sim (8\pi\rho c^2\epsilon_B)^{1/2}[3E/(4\pi\rho c^2)]^{1/3} \\ &\sim 10^{19}\epsilon_{B,-1}n_0^{1/6}E_{51}^{1/3}{\rm eV}, \end{aligned} \tag{11}$$

where we have taken, as an example, a gamma-ray burst source with $E = 10^{51}E_{51}$ erg, $\rho = n\, m_p$ is the ambient density with $n = n_0$ cm^{-3}, and $\epsilon_B = 0.1\epsilon_{B,-1}$. The spectral index above this energy will be $-2 - x_{\rm esc} = -2 + 5x$, as given by Eq. (10). We note that the energy range where $x_{\rm esc} = -5x$ is limited between ε_* and $\max(\varepsilon) \sim \varepsilon_{\rm max}(\Gamma_0)$ where Γ_0 is the initial Lorentz factor. The maximal energy scales with Γ as $\varepsilon_{\rm max} \propto \Gamma^{1/3}$ resulting in a narrow energy range $\varepsilon_* \lesssim \varepsilon \lesssim \Gamma_0^{1/3}\varepsilon_*$. While this range depends on model parameters, it does point to a possible mechanism for achieving a hardening of the spectrum at the high end of the CR spectrum.

We conclude that the spectrum of UHECRs, may be different than ε^{-2}, and possibly considerably harder, e.g. if they originate from a relativistic decelerating blast-wave resulting from the explosion of a stellar mass object.

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TABLE 1
Spectral indexes of escaping CRs for different relativistic scenarios

Scenario	Conserved quantity	α_Γ	$x_{\rm esc}$ $[\rho \propto R^0]$	$x_{\rm esc}$ $[\rho \propto R^{-2}]$
Impulsive, Energy conserving	$E \sim \Gamma^2 \rho R^3$	$(3-\alpha_\rho)/2$	$-5x$	$-x$
Impulsive, Radiative	$M\Gamma \sim \Gamma\rho R^3$	$3-\alpha_\rho$	$-3/2-2x$	$-1/2-x$
Continuous injection L=const.	$L \sim 4\pi\rho\Gamma^4 c^3 R^2$	$(2-\alpha_\rho)/4$	-4+3x	2-x

The value of the spectral index $x_{\rm esc}$ is given by Eqs. (A5) and (A6) for the various scenarios. $N(\varepsilon) \propto \varepsilon^{-2-x_{\rm esc}}$.

APPENDIX

OTHER PARTICULAR CASES

Consider an ultra-relativistic shock, which expands into a medium with a density profile

$$\rho \propto R^{-\alpha_\rho} \tag{A1}$$

with a bulk Lorentz factor

$$\Gamma \propto R^{-\alpha_\Gamma}. \tag{A2}$$

Using Eqs. (7)-(9) and (A1) we find:

$$\alpha_{\rm min} = 2\alpha_\Gamma,\ \ \alpha_{\rm max} = \alpha_\Gamma + \frac{1}{2}\alpha_\rho - 1,\ \text{and}\ \ \alpha_E = 2\alpha_\Gamma + \alpha_\rho - 3. \tag{A3}$$

Substituting this in equation (5) we find

$$x_{\rm esc} = 3 - 2\alpha_\Gamma - \alpha_\rho + \frac{(2-\alpha_\rho)+2\alpha_\Gamma}{(2-\alpha_\rho)-2\alpha_\Gamma}x. \tag{A4}$$

For a uniform density distribution, $\alpha_\rho = 0$, and for a wind-like density distribution, $\alpha_\rho = 2$, this reduces to

$$x_{\rm esc} = \frac{3-2\alpha_\Gamma}{\alpha_\Gamma - 1} + \frac{1+\alpha_\Gamma}{1-\alpha_\Gamma}x, \tag{A5}$$

and

$$x_{\rm esc} = \frac{1-2\alpha_\Gamma}{\alpha_\Gamma + 1} - x \tag{A6}$$

respectively. The escaping spectrum is then

$$N_{esc}(\varepsilon) \propto \varepsilon^{-2-x_{esc}}, \tag{A7}$$

which is flatter then -2 for $x_{esc} < 0$. The expected spectral indices $x_{\rm esc}$, for various physical scenarios, are provided in table 1.